An Iterative Method for Obtaining the Optimum Lightning Location on A Spherical Surface

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ABSTRACT

According to Orville, briefly introduced the basic principle of eigent technique for Obtaining the optimum source location using multiple direction finders on a spherical surface. This technique taking the distance of source-DFs as a constant should be improved. Pointed out that using a weight factor of signal strength is not a idealest method because of the inexact inverse signal strength-distance relation and the inaccurate signal amplitude. Presented an iterative calculation method using distance from source to DF as a weight factor. This method has higher accuracy and needs only a little more calculation time. Showed some computer simulations for a 4DFs system so as to show the improvement of location by using the iterative method.

1. INTRODUCTION

Lightning direction-finding networks [1] have been widely used for the purpose of determining cloud-ground lightning locations. For obtaining the optimum source location we have to resolve the problem of doing optimum calculation under presence of site errors and random errors.

Site errors as high as getting to even 15° are systematic errors [2.3] . These errors have been discussed [4. 5]and some methods have been presented to identify and remove them. The remained problem is simplified to do the optimization location only under the presence of random errors. It has also been solved satisfactorily [6.7]. Especially Orville[6] presented an analytical solution in spherical coordinates for the source on the earth's surface. The analytical solution was obtained by reducing the question to an eigenvalue problem. This method is also called symmetric minimization. It needs only very short time for getting the optimum position using all DF's data. Therefore the eigen technique is very suitable for real time operation.

For mathematical tractability Orville suggested all the distances between source and DFs are the same. This approximation will bring some errors into results. For improving the optimization accuracy, he introduced a method that used a signal strength, which varies inversely with the range from a DF to the flash, to weight the related elements. But it is not a best method because of the unregular properties of signal strength. We will present a simple iterative method based on Orville's eigen technique. We will also introduce some simulation examples for comparation.

2.SYMMETRIC MINIMIZATION

We assume there are no site errors in observations. In Fig. 1,a hypothetical ground flash occurs at point $P(\sigma,\Omega)$ where σ and Ω are latitude and longi-tude of the flash location respectively. A direction finder DFi at (σ_i,Ω_i) detects the point lightning flash at an angle from the true azimuth, at a distance of δ_i to the flash. The great-circle distance from the flash to the bearing line of DFi is hi. We know

$$Sin\ \alpha_i\ =\ Sin\ h_i/Sin\delta_i \qquad (1)$$

For getting the optimum flash location we need to minimize

$$\sum_{i=1}^{n}Sin^2\alpha_i=\sum_{i=1}^{n}Sin^2h_i/Sin^2\delta_i \qquad (2)$$

where n is the total number of DFs. For mathematical tractability Orville proposed to minimize ΣSin^2h_i instead of $Sin^2\alpha_i$ and presented an eigen technique, which is quite elegant and fast. But the approximation that let i=constant will bring some errors to the results.Although it is pointed out that the signal strength reported by DF can used as a weight factor, its improvement is still limited because of the inexact inverse relation between and signal strength or the bigger measuring error of signal strength (up to 20%).

3.ITERATIVE METHOD

First based on Orville's eigen method [6] , let δ_i=constant, we can get the first location approximation (σ,Ω).Then using the relation

$$Cos\delta i=Sin\sigma i*Sin\sigma+Cos\sigma i*Cos\sigma*Cos(\Omega-\Omega i) \qquad (3)$$

we can get the approximate δi. Put the new δi into (2) to find the minimum value

$$\frac{\partial}{\partial \sigma} \Sigma \sin^2 \mu_i = \frac{\partial}{\partial \sigma} \frac{\sin^2 h_i}{\sin^2 \delta_i} \tag{4}$$

$$\frac{\partial}{\partial \Omega} \Sigma \sin^2 \mu_i = \frac{\partial}{\partial \Omega} \frac{\sin^2 h_i}{\sin^2 \delta_i} \tag{5}$$

Here Sin i is a known value. Let a weight factor

$$W_i = 1/\sin \delta i \tag{6}$$

we can obtain following new eigents:

$$\hat{X}_1 = \begin{pmatrix} W_1 (x_1)_1 & W_2 (x_1)_2 & \ldots W_n (x_1)_n \\ W_1 (y_1)_1 & W_2 (y_1)_2 & \ldots W_n (y_1)_n \\ W_1 (z_1)_1 & W_2 (z_1)_2 & \ldots W_n (z_1)_n \end{pmatrix} \tag{7}$$

$$\hat{A}_1 = \begin{pmatrix} \Sigma W_i^2 (x_1)_i^2 & \Sigma W_i^2 (x_1)_i (y_1)_i & \Sigma W (x_1)_i (z_1)_i \\ \Sigma W_i^2 (y_1)_i (x_1)_i & \Sigma W_i^2 (y_1)_i^2 & \Sigma W (y_1)_i (z_1)_i \\ \Sigma W_i^2 (z_1)_i (x_1)_i & \Sigma W_i^2 (z_1)_i (y_1)_i & \Sigma W (z_1)_i^2 \end{pmatrix} \tag{8}$$

$$\hat{X}_3 = \begin{pmatrix} W_1 (x_3)_1 & W_2 (x_3)_2 & \ldots W_n (x_3)_n \\ W_1 (y_3)_1 & W_2 (y_3)_2 & \ldots W_n (y_3)_n \\ W_1 (z_3)_1 & W_2 (z_3)_2 & \ldots W_n (z_3)_n \end{pmatrix} \tag{9}$$

$$\hat{A}_3 = \begin{pmatrix} \Sigma W_i^2 (x_3)_i^2 & \Sigma W_i^2 (x_3)_i (y_3)_i & \Sigma W_i^2 (x_3)(z_3)_i \\ \Sigma W_i^2 (y_3)_i (x_3)_i & \Sigma W_i^2 (y_3)_i^2 & \Sigma W_i^2 (y_3)(z_3)_i \\ \Sigma W_i^2 (z_3)_i (x_3)_i & \Sigma W_i^2 (z_3)_i (y_3)_i & \Sigma W_i^2 (z_3) \end{pmatrix} \tag{10}$$

The difference between eigents above and those of Orville's is that each term above is weighted by factor W_i.

The following formulas should be same as Orville's:

$$\hat{R}_1 \hat{A}_1 = \hat{A}_3 \hat{R} \tag{11}$$

$$\hat{A}_3 = \begin{pmatrix} w & u & 0 \\ u & v & 0 \\ 0 & 0 & r \end{pmatrix} \tag{12}$$

$$\vec{V}_3 \hat{A}_1 = r \vec{V}_3 \tag{13}$$

Using the same calculation procedure we will obtain the iterated results, each is the new optimum probable location ($\mathcal{G}$,Ω). Of course we can do the iterative procedure several times so as to reach higher accuracy, but we have also found it is perfectly enough for general location to do iterative only once.

4.RESULTS COMPARATION

For understanding the improvement of using iterative calculation, we have done some computer simulation. Assume there is a 4DFs network on the earth's spherical surface roughly in a square geometric figure with the center at latitude 39 N and longitude 115 E. The baseline of adjacent DF is about 90KM and the discussed area is about 200*200KM. For any arbitrary point in this area, we know its exact direction related to every DF. Add 300 random errors to the ideal directions in an angle range of 1 . Using two algorithms: unweighted symmetric minimization and iterative method respectively, we can get two sets of 300 new simulated lightning locations(to do iterative once) and further the distances between them and the ideal point. Take average of two sets distances we will obtain the statistics location error related to two methods. Repeat this simulation at different point(400 points) in whole area and compare their location errors, the improvement of iterative calculation will be clearly showed on Fig. 2-Fig. 4. Generally saying, iterative method will decrease the location error by 20% and extend the equal precision area much more.

For discussing the necessary iterative times we have tested iteration four times and compared the distance from center to the calculation points. Table I shows some examples of ten points and from the results we can conclude that once iteration is enough for acceptible accuracy.

5. DISCUSSION

An iterative method for getting optimum lightning location has been presented. This method is an improvement on Orville's eigent method. It needs only the first distance opproximation of flash to DF as a weight factor to repeat the eigent technique once more. So it spends only a little more time during the real time calculation,but gets more accurate location result.
This technique has been used in a lightning location network around Beijing and we are satisfied with it.

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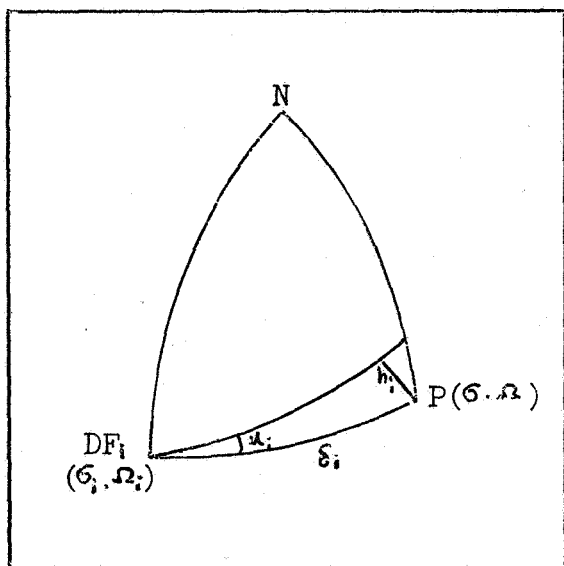
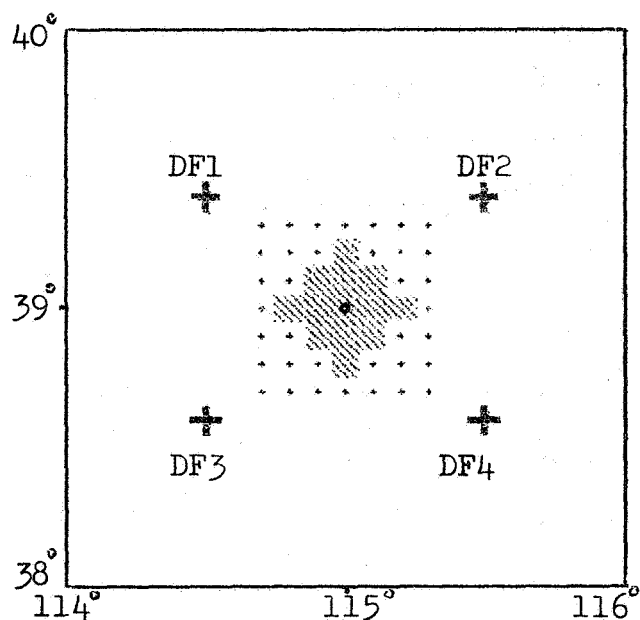Fig. 1. Spherical sketch geometry for optimum flash location.

Fig. 2. Simulation results with 0.6KM location accuracy. Shadow represents the area using symmytric technique, dots are the expended area by iterative method.
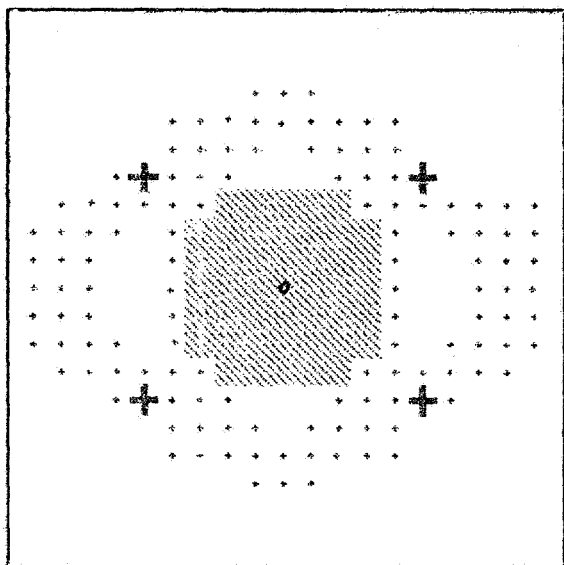
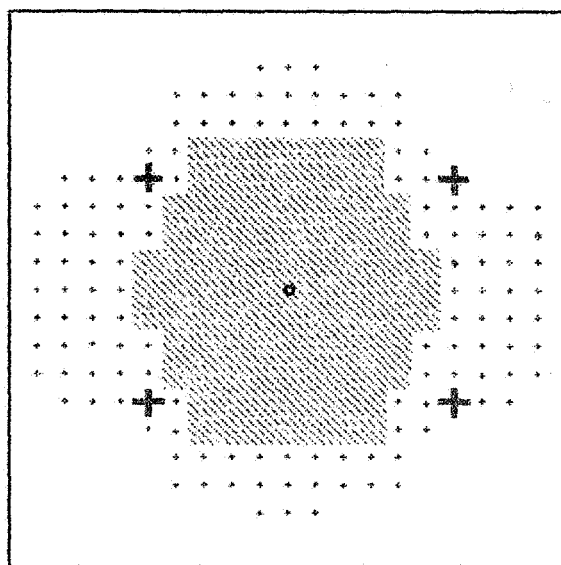Fig. 3. Same as Fig. 2, but with 0.7KM location accuracy.

Fig. 4. Same as Fig. 2, but with 0.8KM location accuracy.

Table I. Comparison of the source-centre distance by using
different calculation methods.

Source	Calculation Method					
Number	True	Symme-tric	Interative			
			1st	2ed	3rd	4th
1	127.49	128.27	127.50	127.49	127.49	127.49
2	99.78	100.11	99.78	99.79	99.78	99.78
3	124.77	124.67	124.77	124.77	124.77	124.77
4	78.61	78.65	78.61	78.61	78.61	78.61
5	0.60	0.59	0.59	0.60	0.60	0.60
6	77.48	76.99	77.48	77.48	77.48	77.48
7	128.55	129.05	128.55	128.56	128.55	128.55
8	99.79	100.11	99.79	99.79	99.79	99.79
9	125.02	125.14	125.02	125.02	125.03	125.03
10	33.66	33.63	33.66	33.66	33.66	33.66